Neutron holography without reference beam

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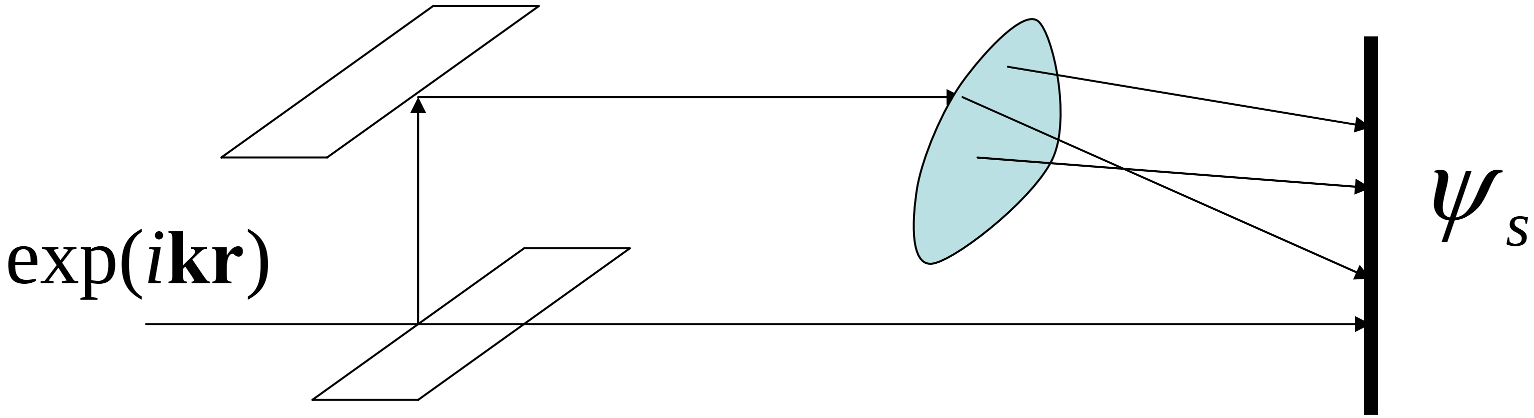
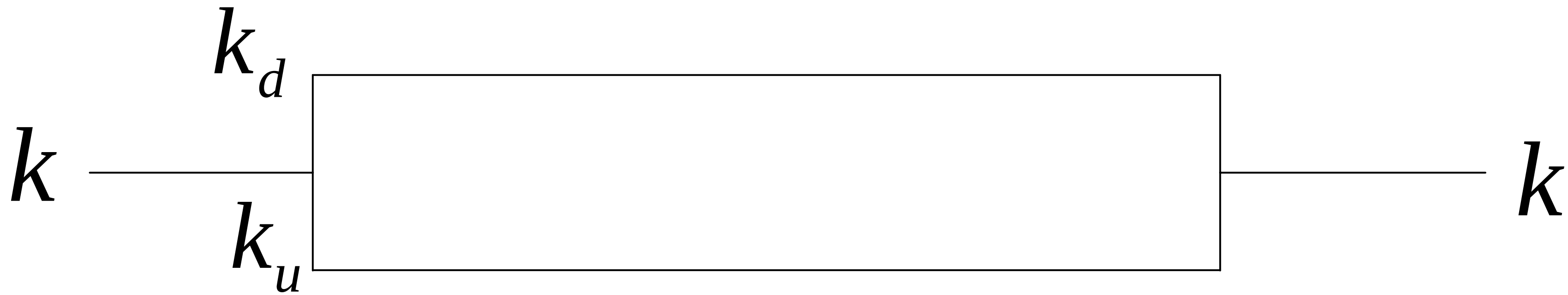
PSI 14.02.2006

Principles of holography



$$I(\mathbf{r}) = \left|\exp(i\mathbf{kr}) + \psi_s\right|^2 \approx 1 + \exp(i\mathbf{kr})\psi_s^* + \exp(-i\mathbf{kr})\psi_s$$

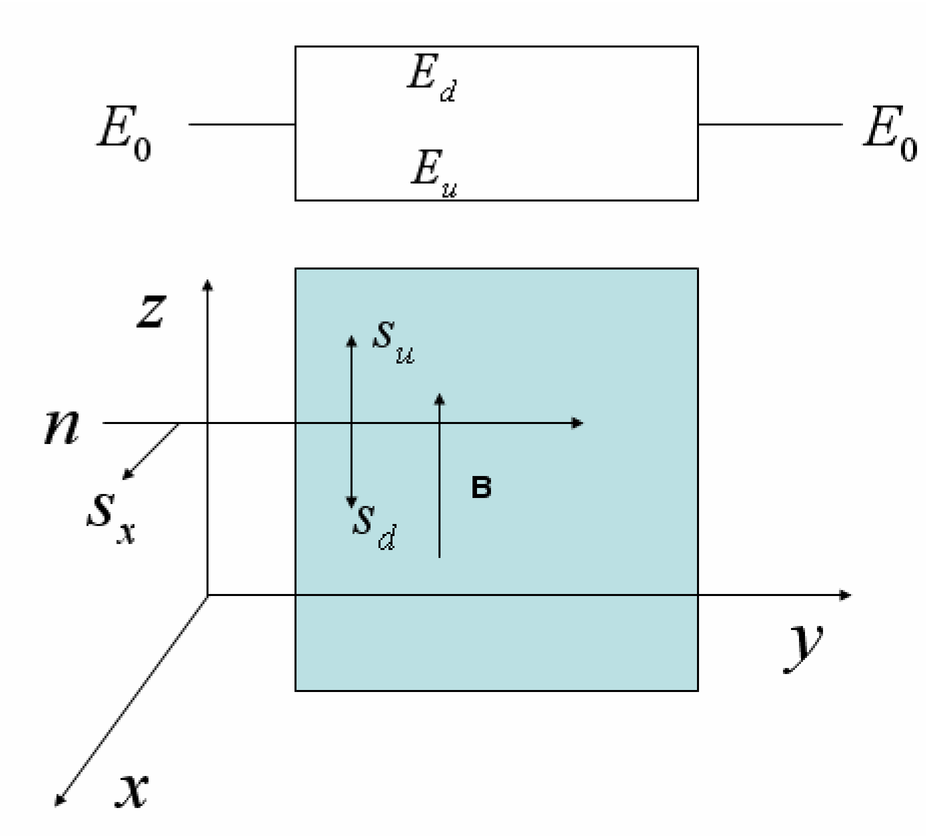
Neutron holography without reference beam



$$\psi_s = b\,\frac{\exp(ik_1 r) + \exp(ik_2 r)}{\sqrt{2}r}$$

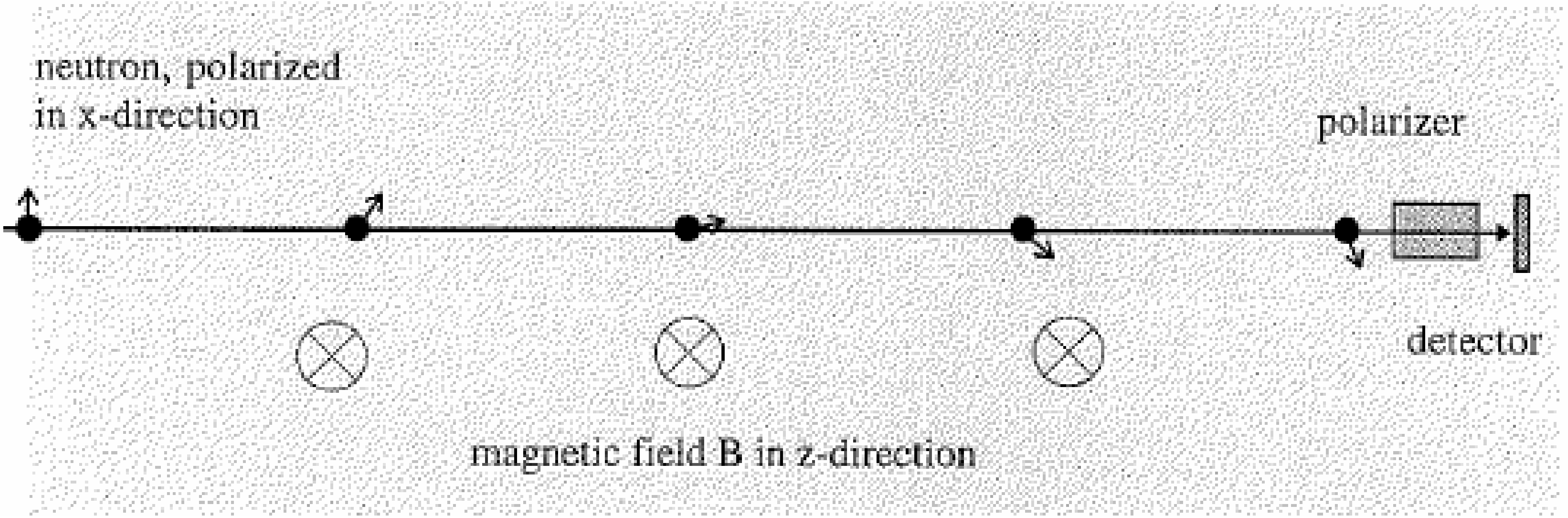
$$I_s = |b|^2\,\frac{1 + \cos((k_1 - k_2)r)}{r^2}$$

Spin precession in magnetic field



$$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}=\frac{1}{\sqrt{2}}\left(\begin{pmatrix}1\\0\end{pmatrix}+\begin{pmatrix}0\\1\end{pmatrix}\right)=\frac{1}{\sqrt{2}}\left(\exp(ik_u x)\begin{pmatrix}1\\0\end{pmatrix}+\exp(ik_d x)\begin{pmatrix}0\\1\end{pmatrix}\right)$$

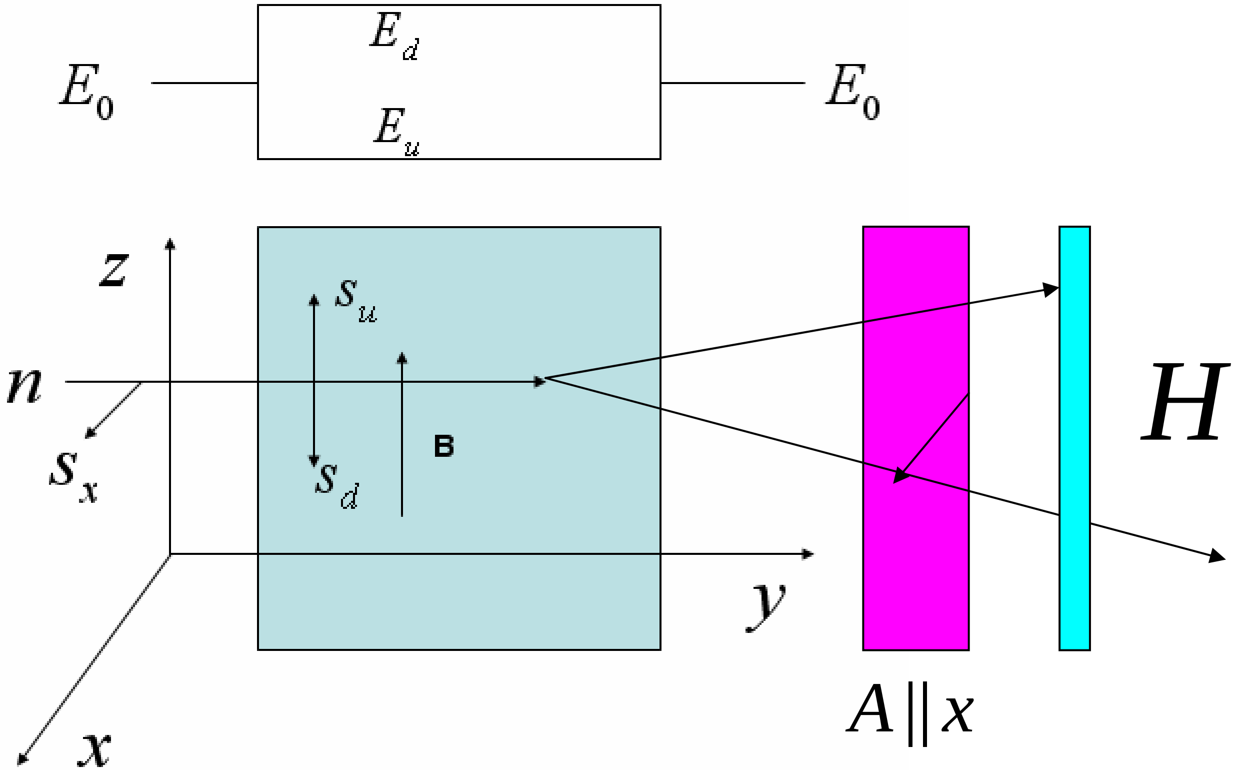
Neutron spin optics in magnetic field



$$|\xi_{xu}\rangle = \frac{1}{\sqrt{2}}\left(|\xi_{zu}\rangle + |\xi_{zd}\rangle\right)$$

$$|\psi(r)\rangle = \frac{1}{\sqrt{2}}\exp\left(-iE_0 t + ir\sqrt{k_0^2 - \omega\sigma_z}\right)\left(|\xi_{zu}\rangle + |\xi_{zd}\rangle\right) =$$

$$= \frac{1}{\sqrt{2}}\left(\exp\left(-iE_0 t + ir\sqrt{k_0^2 - \omega}\right)|\xi_{zu}\rangle + \exp\left(-iE_0 t + ir\sqrt{k_0^2 + \omega}\right)|\xi_{zd}\rangle\right)$$



$$k_u = \sqrt{k^2 - 2B}$$

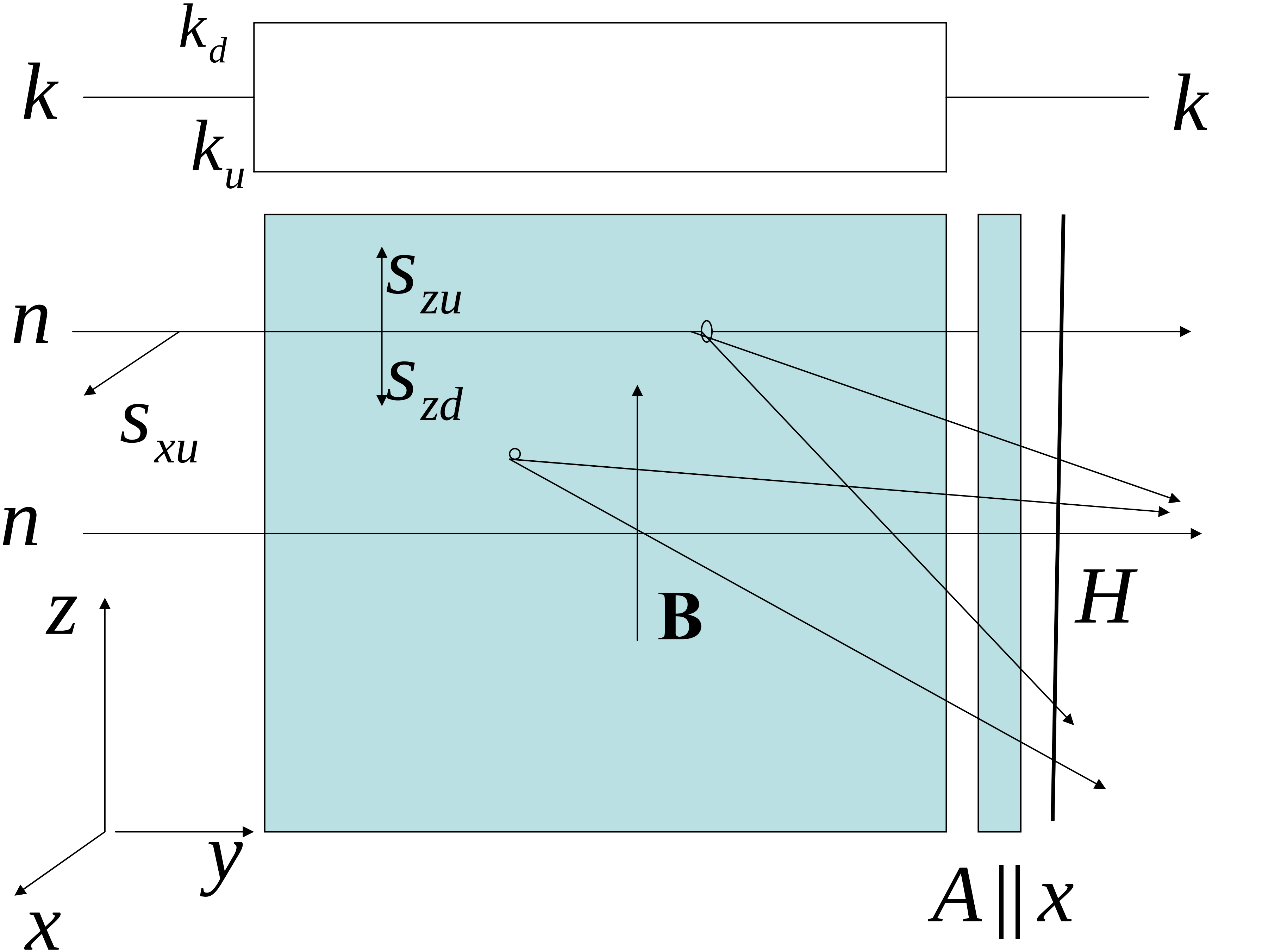
$$k_d = \sqrt{k^2 + 2B}$$

$$k\_ \approx k\frac{B}{k^2}$$

$$\frac{\exp(ik_+x)}{\sqrt{2}}\left(\exp(-ik\_x)\begin{pmatrix}1\\0\end{pmatrix}+\exp(ik\_x)\begin{pmatrix}0\\1\end{pmatrix}\right)\Rightarrow\frac{\exp(ik_+x)}{\sqrt{2}}\cos(k\_x)\begin{pmatrix}1\\1\end{pmatrix}$$

$$k_\pm = \frac{1}{2}(k_d \pm k_u)$$

$$I(x) = \cos^2(k\_x)$$



k_d
k
k
k_u
s_{zu}
n
s_{zd}
s_{xu}
n
H
z
$\mathbf{B}$
y
$A \parallel x$
x

Estimation of wavelength

$$I(x) \propto 2\cos^2(k\_x) = 1 + \cos(2k\_x)$$

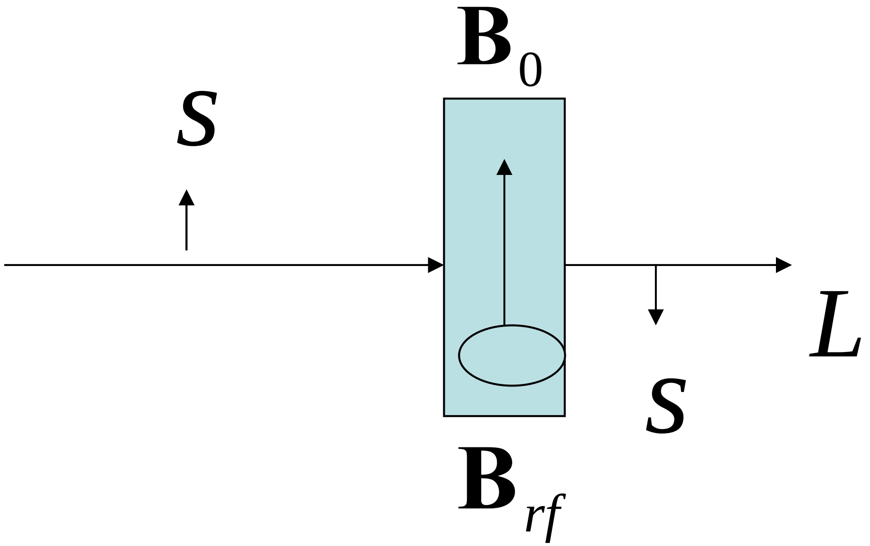
$$2k_{\_} \approx k\frac{2B}{k^2} \qquad \Lambda = \frac{2\pi}{2k_{\_}} = \frac{\lambda_c^2}{\lambda} = 6000\,\mathring{A}$$

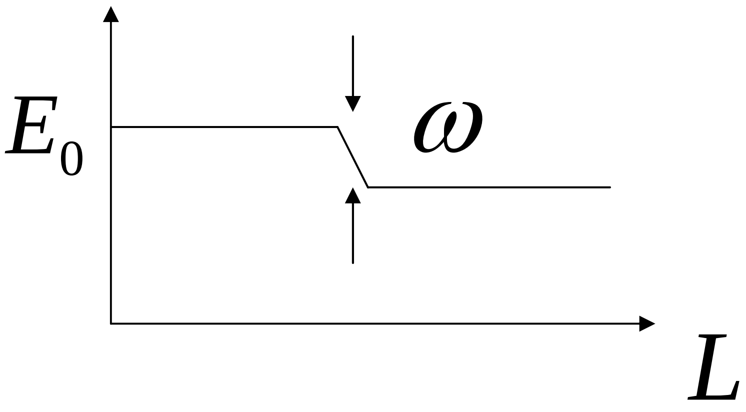
$$\lambda_c = \frac{2\pi}{\sqrt{2B}} \qquad \lambda = \frac{\lambda_c^2}{\Lambda}$$

$$B = 1\,\mathrm{kGs} \Rightarrow \lambda_c \approx 2000\,\mathring{A} \Rightarrow \lambda \approx 600\,\mathring{A}$$

UCN neutrons

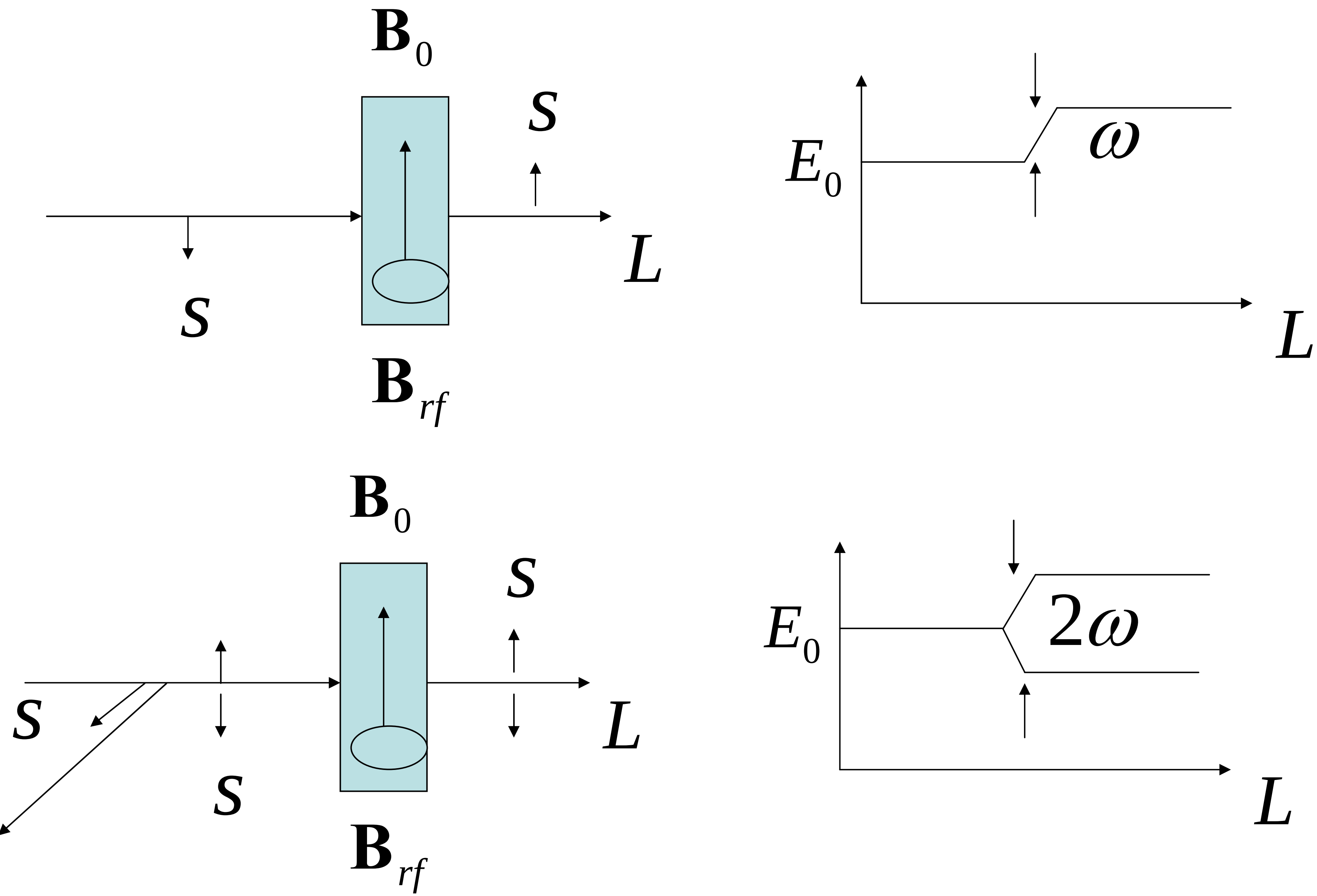
Spin-flip and no external field





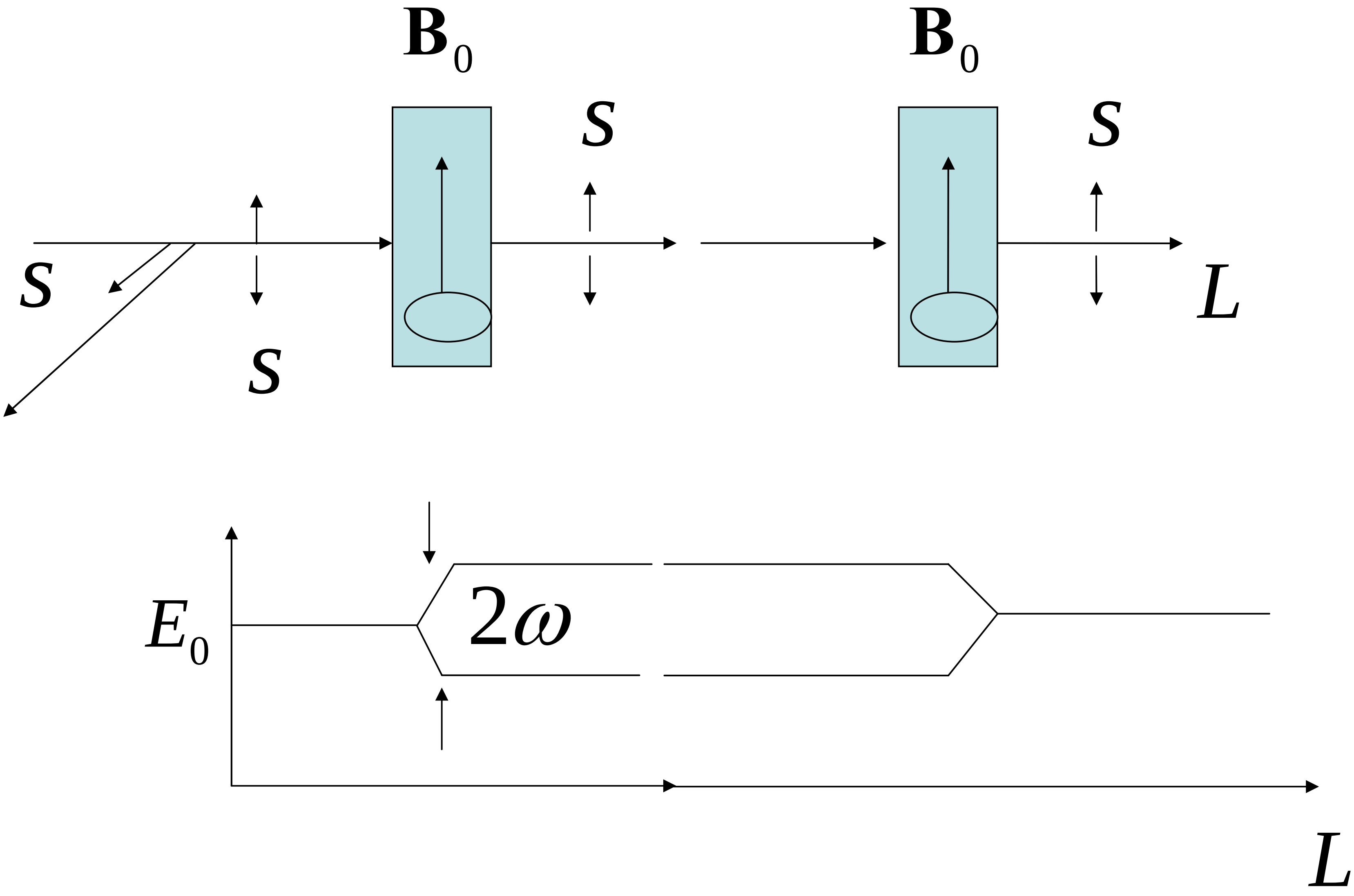
$$w_{\downarrow\uparrow} = \frac{B_{rf}^2}{(\omega - B_0)^2 + B_{rf}^2}\sin^2\left(\sqrt{(\omega - B_0)^2 + B_{rf}^2}\,T\right)$$

Spin-flip



$\mathbf{B}_0$
S
S
L
$\mathbf{B}_{rf}$
E_0
ω
L
$\mathbf{B}_0$
S
S
S
L
$\mathbf{B}_{rf}$
E_0
2ω
L

2 identical flippers



Wave-function

Before flipper

$$|\Psi_0(y,t)\rangle = \exp(ik_0 y - iE_0 t)\,|\,\xi_{xu}\rangle =$$

$$= \exp(ik_0 y - iE_0 t)\frac{1}{\sqrt{2}}\left[|\,\xi_{zu}\rangle + |\,\xi_{zd}\rangle\right]$$

After flipper

$$|\Psi_1(y,t)\rangle = \frac{-i}{\sqrt{2}}\exp(i\sqrt{k_0^2 + 2\omega\sigma_z}\,y - i(E_0 + \omega\sigma_z)t)\left[|\,\xi_{zd}\rangle + |\,\xi_{zu}\rangle\right]$$

$$= \frac{-i}{\sqrt{2}}\exp(i\sqrt{k_0^2 - 2\omega}\,y - i(E_0 - \omega)t)\,|\,\xi_{zd}\rangle +$$

$$+ \frac{-i}{\sqrt{2}}\exp(i\sqrt{k_0^2 + 2\omega}\,y - i(E_0 + \omega)t)\,|\,\xi_{zu}\rangle$$

Estimation of wavelength

$$I(x) \propto 2\cos^2(k\_x) = 1 + \cos(2k\_x)$$

$$2k\_ \approx k\frac{2B}{k^2} = k\frac{\omega}{k^2}$$

$$\Lambda = \frac{2\pi}{2k\_} = \frac{\lambda_c^2}{\lambda} = 6000\,\overset{o}{\mathrm{A}}$$

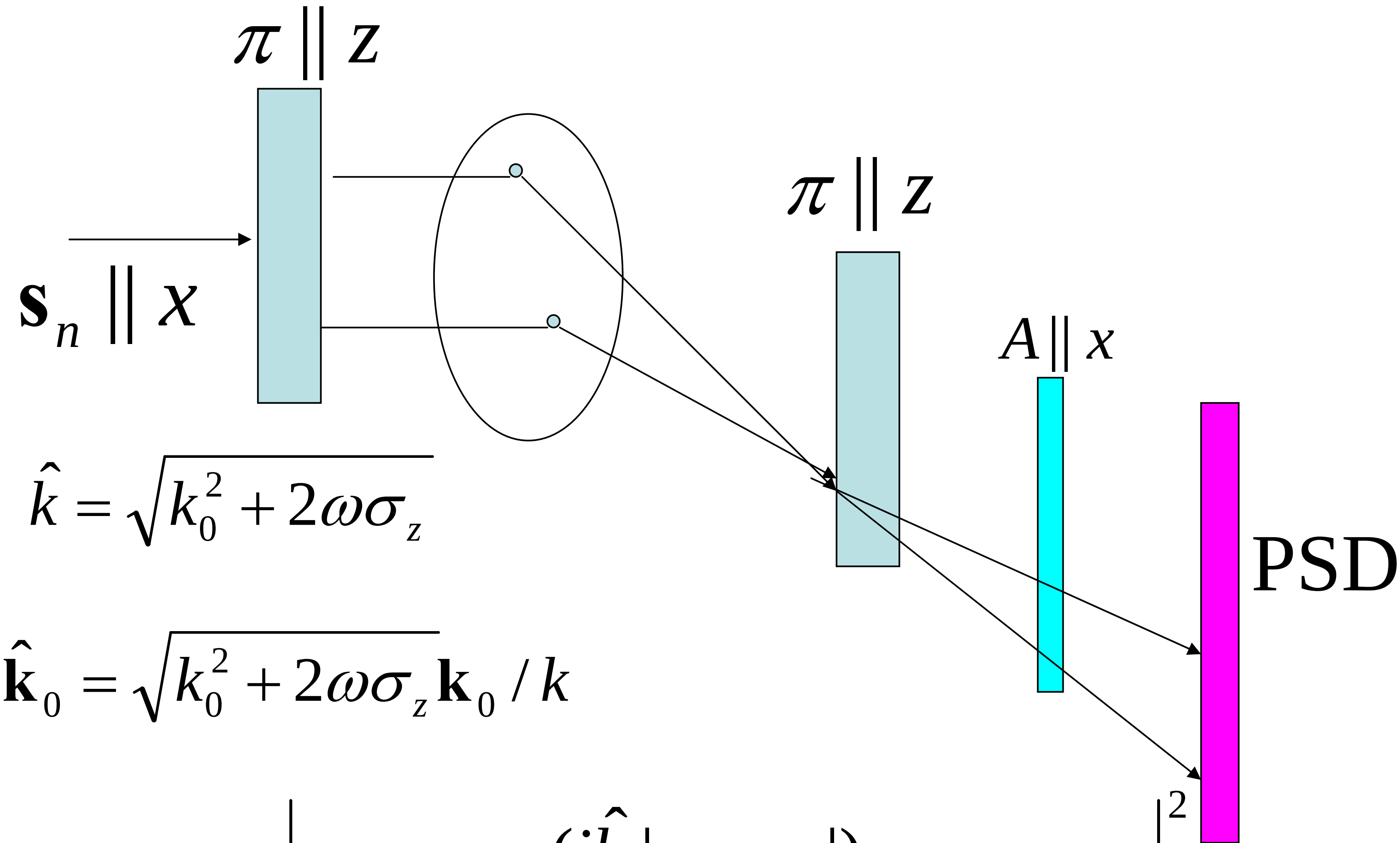
$$\lambda_c = \frac{2\pi}{\sqrt{2B}}$$

$$\lambda = \frac{\lambda_c^2}{\Lambda}$$

$$B = 30\,\mathrm{kGs} \Rightarrow \lambda_c \approx 400\,\overset{o}{\mathrm{A}} \Rightarrow \lambda \approx 30\,\overset{o}{\mathrm{A}}$$

$$\mathrm{v} \approx 200\ \mathrm{m/s} \Rightarrow \mathrm{E} \approx 0.2\,\mathrm{meV}$$

Scattered incoherent waves



$$\hat{k} = \sqrt{k_0^2 + 2\omega\sigma_z}$$

$$\hat{\mathbf{k}}_0 = \sqrt{k_0^2 + 2\omega\sigma_z}\,\mathbf{k}_0 / k$$

$$I(\mathbf{r}) = \sum_j \left|b_j\right|^2 \left| \langle \xi_{xu} \,|\, \frac{\exp(i\hat{k}\,|\mathbf{r}-\mathbf{r}_j|)}{|\mathbf{r}-\mathbf{r}_j|} e^{i\hat{\mathbf{k}}_0\mathbf{r}_j} \,|\, \xi_{xu} \rangle \right|^2$$

No spherical, only plane waves

$$\psi = \exp(i\mathbf{kr}) - \frac{b}{r}e^{ikr} = \exp(i\mathbf{kr}) - i\frac{bk}{2\pi}\int \exp(i\mathbf{k}_{\Omega}\mathbf{r})d\Omega$$

$$|\mathbf{k}_{\Omega}| = |\mathbf{k}| = k$$

$$\frac{\exp(ikr)}{r} = \frac{4\pi}{(2\pi)^3}\int \frac{d^3p\exp(i\mathbf{pr})}{p^2 - k^2 - i\varepsilon} =$$

$$= \frac{i}{2\pi}\int \frac{d^2p_{\perp}}{\sqrt{k^2 - p_{\perp}^2}}\exp\left(i\mathbf{p}_{\perp}\mathbf{r}_{\perp} + i\sqrt{k^2 - p_{\perp}^2}\,|z|\right)$$

Neglect exponentially decaying waves

$$\frac{i}{2\pi}\left(\int_{p_\perp^2>k^2} + \int_{p_\perp^2<k^2}\right)\frac{d^2p_\perp}{\sqrt{k^2-p_\perp^2}}\exp\left(i\mathbf{p}_\perp\mathbf{r}_\perp + i\sqrt{k^2-p_\perp^2}\,|z|\right)\approx$$

$$\approx\frac{i}{2\pi}\int_{p_\perp^2<k^2}\frac{d^2p_\perp}{\sqrt{k^2-p_\perp^2}}\exp\left(i\mathbf{p}_\perp\mathbf{r}_\perp + i\sqrt{k^2-p_\perp^2}\,|z|\right)=$$

$$=\frac{i}{\pi}\int_{p_\parallel z>0} d^3p\,\delta(p^2-k^2)\exp\left(i\mathbf{pr}\right)=$$

$$\frac{d^2p_\perp}{2p_\parallel} = d^3p\,\delta(p^2-k^2)$$

$$=ik\int_{2\pi}\frac{d\Omega}{2\pi}\exp(i\mathbf{k}_\Omega\mathbf{r})$$

Incoherent hologram

$$I(\mathbf{r}) = \int \frac{d^3\rho d\Omega}{(2\pi)^2} k^2 \Sigma(\boldsymbol{\rho}) \cos^2(\mathbf{q}_0\boldsymbol{\rho} + \mathbf{q}_\Omega(\mathbf{r}-\boldsymbol{\rho})) =$$

$$= \int \frac{d^3\rho d\Omega}{2(2\pi)^2} k^2 \Sigma(\boldsymbol{\rho}) \left[1 + \cos(2\mathbf{q}_0\boldsymbol{\rho} + 2\mathbf{q}_\Omega(\mathbf{r}-\boldsymbol{\rho}))\right] =$$

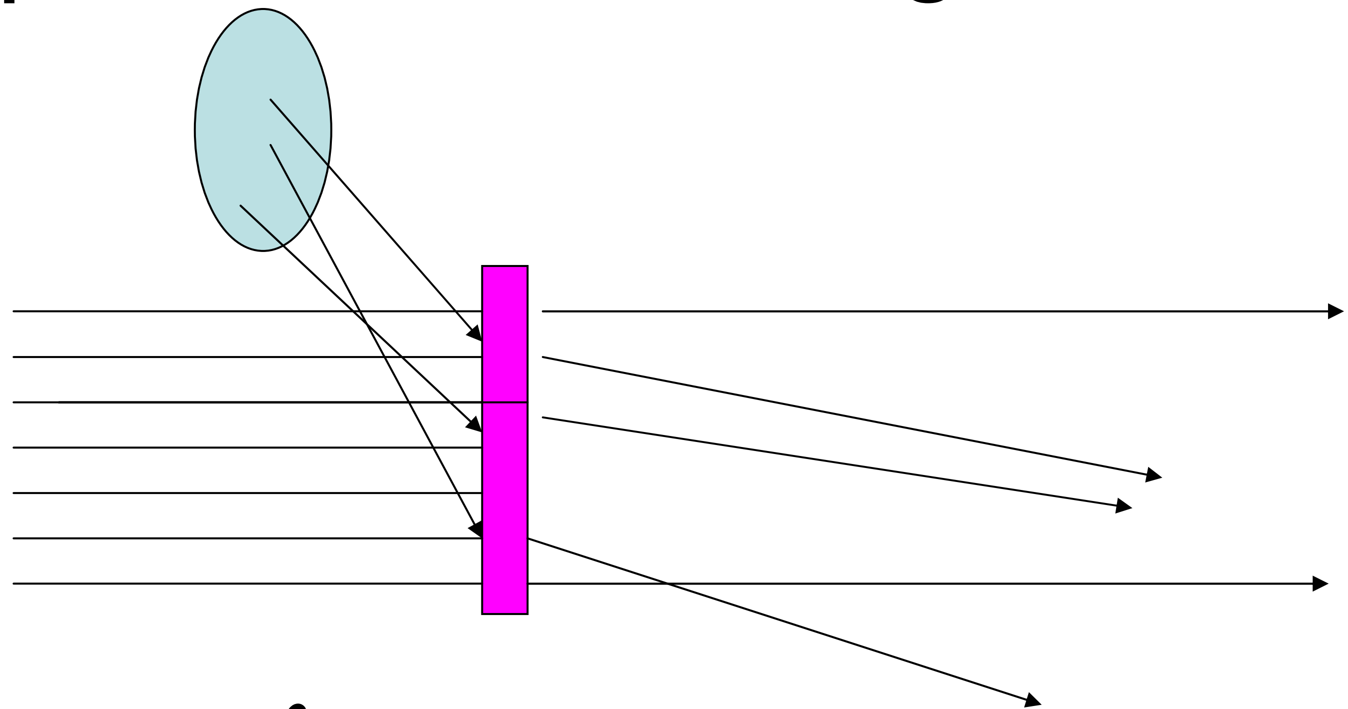
$$= C + \int \frac{d^3\rho d\Omega}{2(2\pi)^2} k^2 \Sigma(\boldsymbol{\rho}) \cos(\mathbf{p}_0\boldsymbol{\rho} + \mathbf{p}_\Omega(\mathbf{r}-\boldsymbol{\rho}))$$

$$C = \int \frac{d^3\rho}{2(2\pi)^2} k^2 \Sigma(\boldsymbol{\rho}) \qquad \mathbf{p}_0 = \mathbf{k}_0 \frac{\omega}{E_0} \qquad \mathbf{p}_\Omega = \mathbf{k}_\Omega \frac{\omega}{E_0}$$

$$\Sigma(\boldsymbol{\rho}) = |b(\boldsymbol{\rho})|^2 \; N_0(\boldsymbol{\rho})$$

Reproduction of image

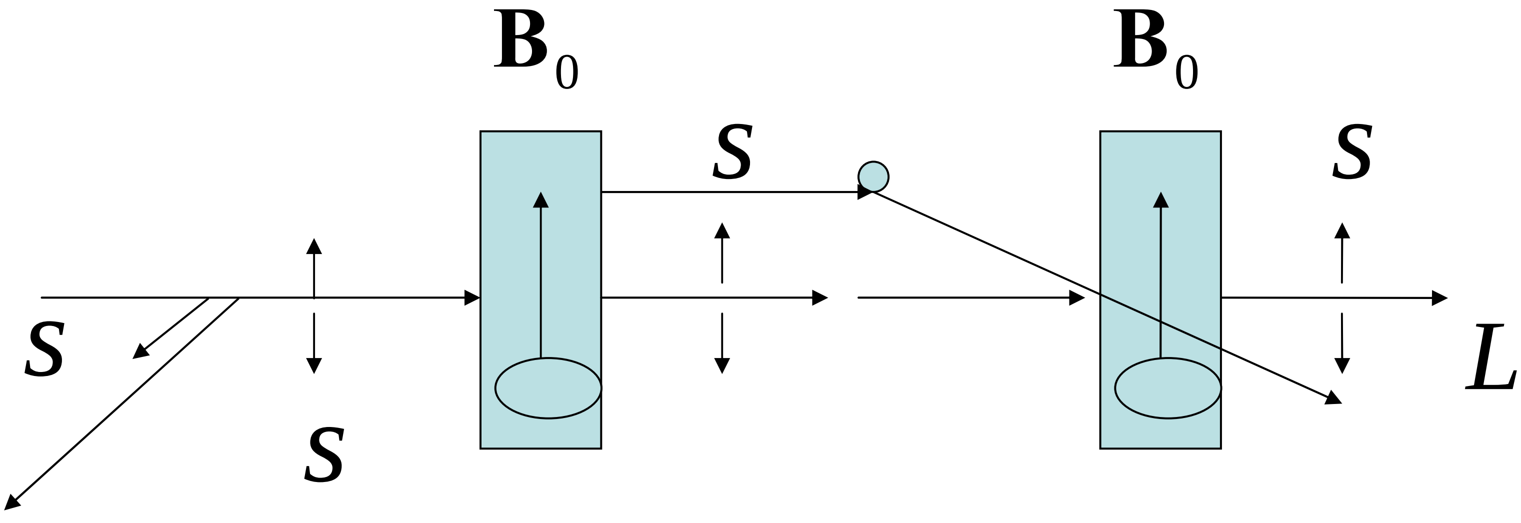
$$e^{i\mathbf{kr}-\nu t}$$

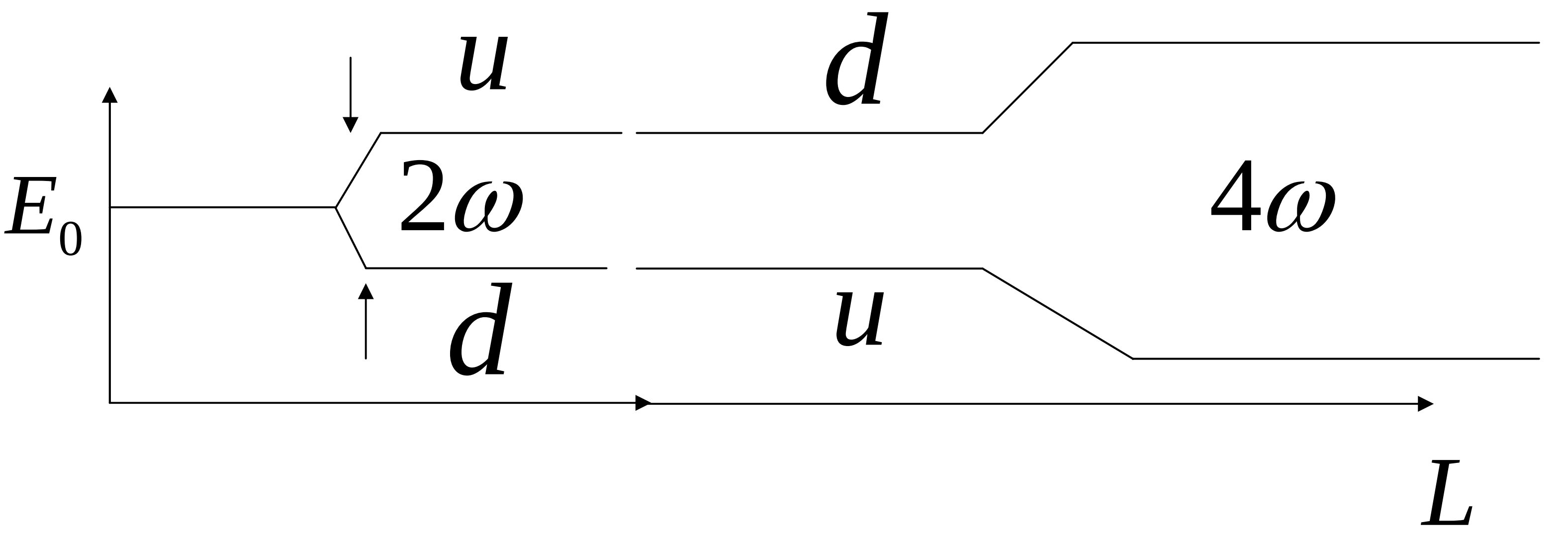


$$\int d^2 l_{\|} e^{i\mathbf{kl}-\nu t} T(\mathbf{l}_{\|}) \int d^2 q_{\|} \exp(i\mathbf{q}(\mathbf{r}-\mathbf{l}))$$

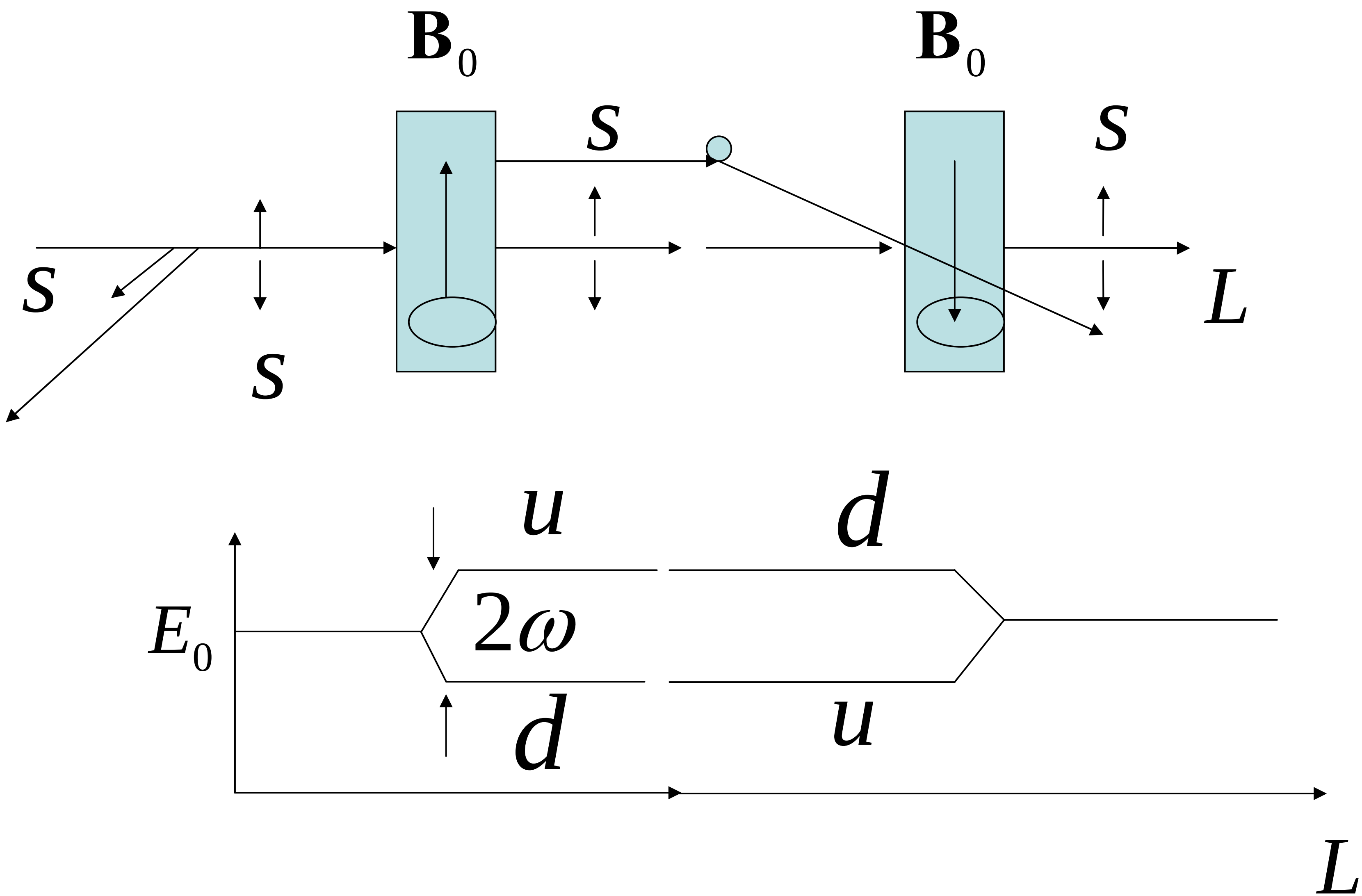
$$T(\mathbf{l}_{\|}) \propto \int \frac{d^3\rho\, d\Omega}{2(2\pi)^2} k^2 \Sigma(\boldsymbol{\rho}) \cos(\mathbf{p}_0\boldsymbol{\rho} + \mathbf{p}_\Omega(\mathbf{l}_{\|} - \boldsymbol{\rho}))$$

Magnetic scattering



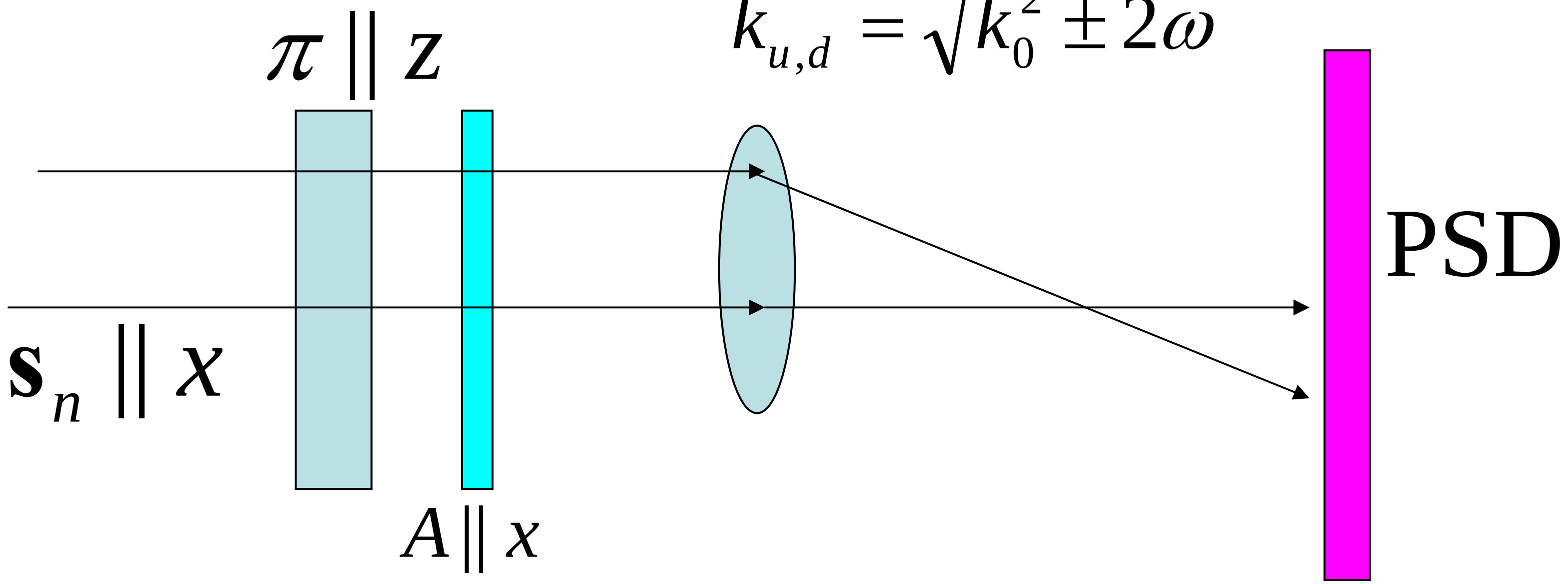


2 identical flippers



Intensity modulated wave

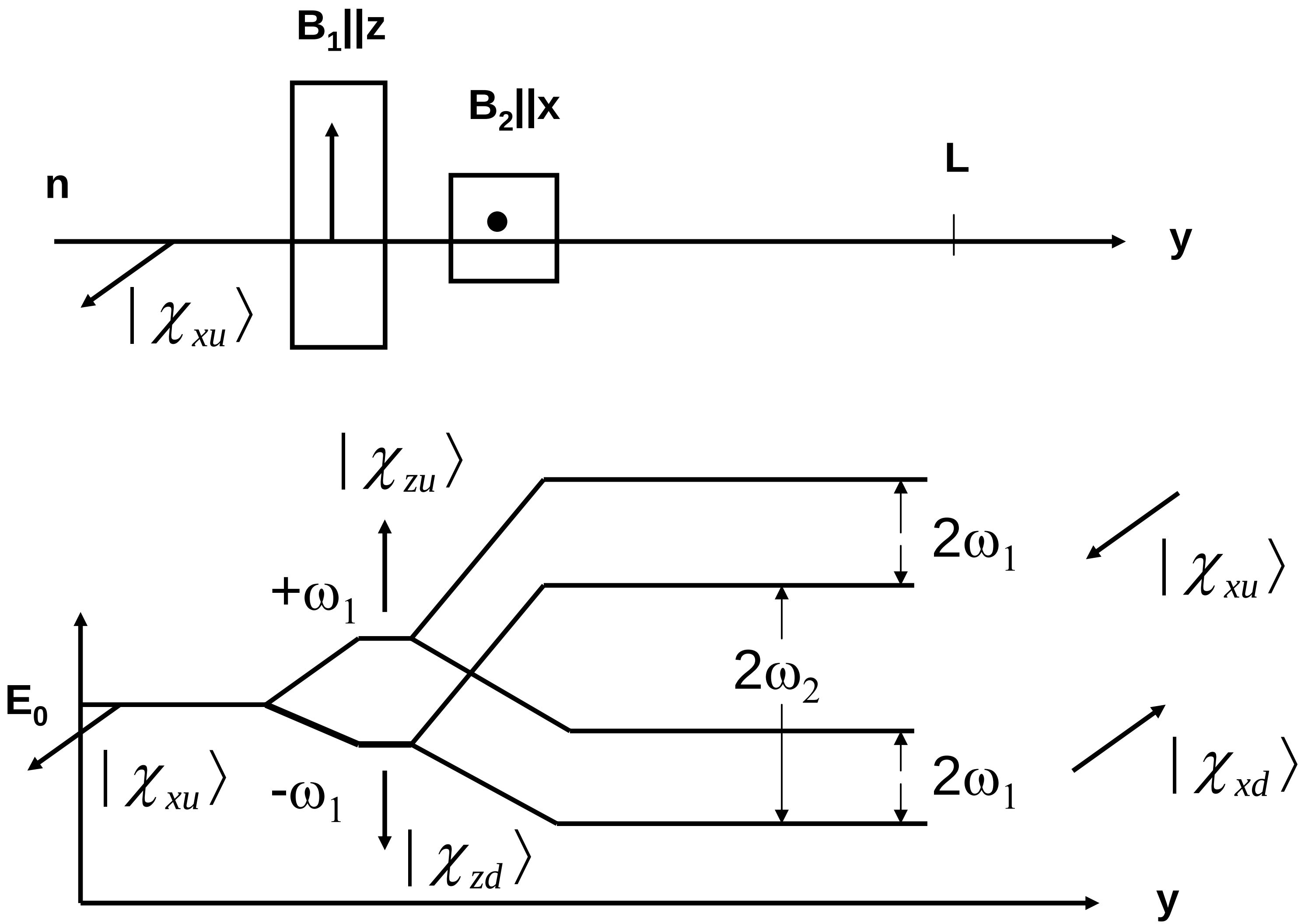
$$k_{u,d} = \sqrt{k_0^2 \pm 2\omega}$$



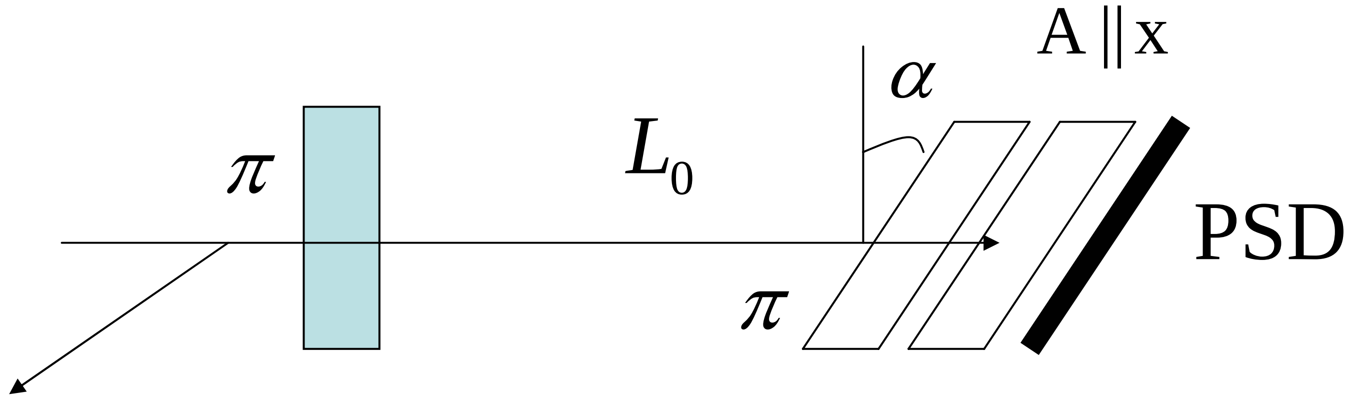
$$\psi = \frac{1}{2}\left\{\exp\left(i\mathbf{k}_u\mathbf{r} - i[E_0 + \omega t]\right) + \exp\left(i\mathbf{k}_d\mathbf{r} - i[E_0 - \omega t]\right)\right\} =$$

$$I(\mathbf{r}, t) = \cos^2(\mathbf{k}\_\mathbf{r} - \omega t)$$

$$k_- = \left(\sqrt{k_0^2 + 2\omega} - \sqrt{k_0^2 - 2\omega}\right)/\,2 \approx \omega / k_0$$


B1||z
B2||x
L
n
y
|χxu⟩
|χzu⟩
+ω1
2ω1
|χxu⟩
E0
2ω2
|χxu⟩
-ω1
2ω1
|χxd⟩
|χzd⟩
y

Two flippers



$$|\Psi_1(y,t)\rangle = \frac{-i}{\sqrt{2}} \exp(i\sqrt{k_0^2 + 2\omega\sigma_z}\, y - i(E_0 + \omega\sigma_z)t)\left[|\xi_{zd}\rangle + |\xi_{zu}\rangle\right]$$

$$|\Psi_2(y,t)\rangle = \frac{-1}{\sqrt{2}} e^{ik_0(y-L)-iE_0 t}\left(e^{i\sqrt{k_0^2+2\omega L}}\,|\xi_{zu}\rangle + e^{i\sqrt{k_0^2-2\omega L}}\,|\xi_{zd}\rangle\right)$$

$$\langle\xi_{xu}\,||\,\Psi_2(y,t)\rangle = \frac{-1}{2} e^{ik_0(y-L)-iE_0 t}\left(e^{i\sqrt{k_0^2+2\omega L}} + e^{i\sqrt{k_0^2-2\omega L}}\right)$$

$$I(L) = \cos^2(\omega L / k_0) = \cos^2(\omega[L_0 + l\sin\alpha]/k_0) = \cos^2(\varphi + l/\Lambda)$$

$$\Lambda = \lambda(2E_0 / \omega \sin\alpha)$$

Conclusion

It is very interesting and feasible!

All the details, like resolution must be considered and will be considered separately