Investigation of the neutron wave packet properties

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The only thing in my life,
In which I am absolutely sure,
is:
that wave packets do not exist!

Roland Gähler

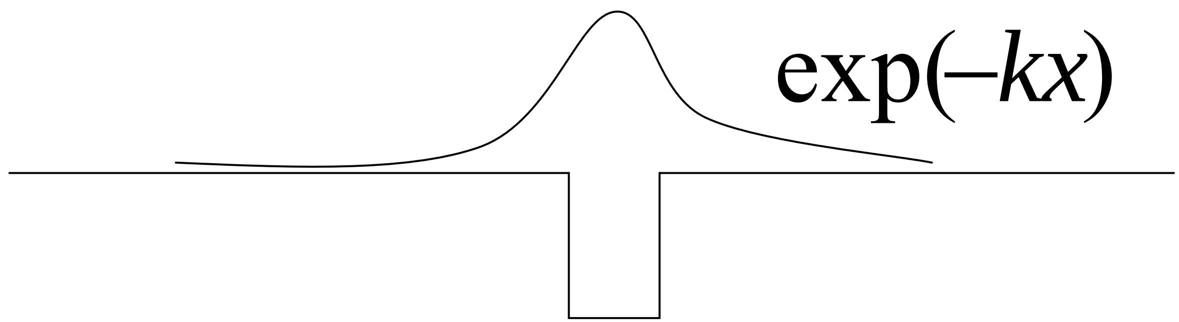
Spherical harmonics

$$\psi = \exp(i\mathbf{kr}) - \frac{b}{r}e^{ikr}$$

Spherical wave does not represent a free particle!

$$(\Delta + k^2)\frac{\exp(ikr)}{r} = -4\pi\delta(\mathbf{r})$$

It does not matter, that it satisfies free equation outside r=0



A bound state wave function also satisfies free equation outside of the well, but nobody tells that w-function tails are free particles!

To get free asymptotic we must exclude exponentially decaying waves

$$\psi = \exp(i\mathbf{kr}) - \frac{b}{r} e^{ikr}$$

$$(\Delta + k^2)\frac{\exp(ikr)}{r} = -4\pi\delta(\mathbf{r})$$

$$\frac{\exp(ikr)}{r} = \frac{4\pi}{(2\pi)^3}\int \frac{d^3 p}{p^2 - k^2 - i\varepsilon} =$$

$$= \frac{i}{2\pi}\int \frac{d^2 p_\perp}{\sqrt{k^2 - p_\perp^2}} \exp\left(i\mathbf{p}_\perp \mathbf{r}_\perp + i\sqrt{k^2 - p_\perp^2}\,|z|\right)$$

Neglect exponentially decaying waves

$$\frac{i}{2\pi}\left(\int_{p_\perp^2>k^2} + \int_{p_\perp^2<k^2}\right)\frac{d^2p_\perp}{\sqrt{k^2-p_\perp^2}}\exp\left(i\mathbf{p}_\perp\mathbf{r}_\perp + i\sqrt{k^2-p_\perp^2}\,|z|\right)\approx$$

$$\approx\frac{i}{2\pi}\int_{p_\perp^2<k^2}\frac{d^2p_\perp}{\sqrt{k^2-p_\perp^2}}\exp\left(i\mathbf{p}_\perp\mathbf{r}_\perp + i\sqrt{k^2-p_\perp^2}\,|z|\right)=$$

$$\frac{i}{\pi}\int_{p_{||}z>0} d^3p\,\delta(p^2-k^2)\exp\left(i\mathbf{pr}\right)=$$

$$\frac{d^2p_\perp}{2p_{||}} = d^3p\,\delta(p^2-k^2)$$

$$= ik\int_{2\pi}\frac{d\Omega}{2\pi}\exp(i\mathbf{k}_\Omega\mathbf{r})$$

Scattering asymptotic of spherical wave

$$\psi = \exp(i\mathbf{kr}) - \frac{b}{r} e^{ikr} \Rightarrow$$

$$\Rightarrow \exp(i\mathbf{kr}) - i\frac{kb}{2\pi}\int_{4\pi} d\Omega \exp(i\mathbf{k}_{\Omega}\mathbf{r})$$

$$|\mathbf{k}_{\Omega}| = k$$

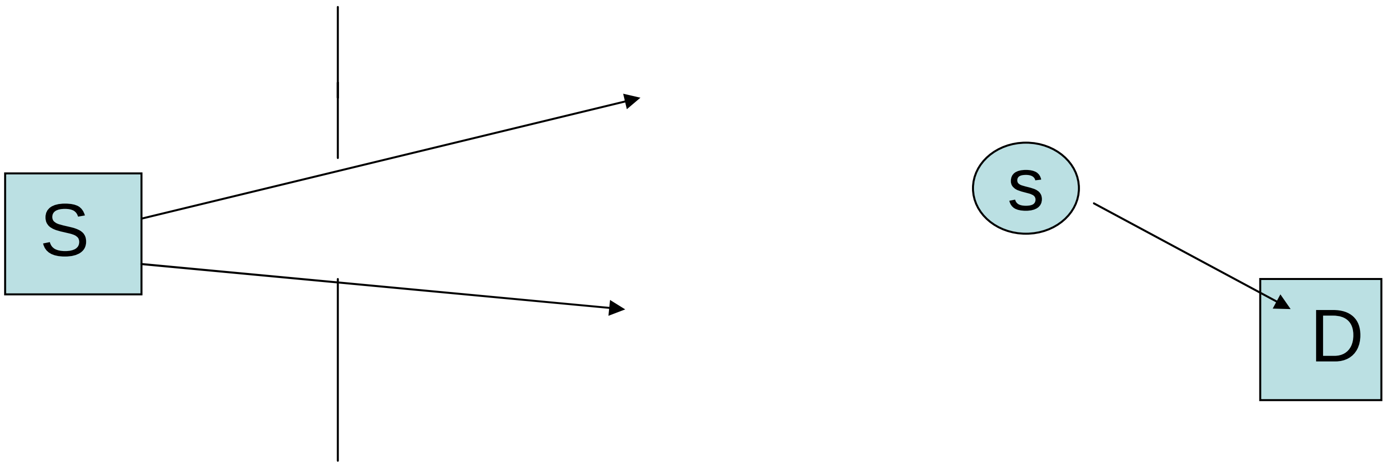
$$dw = \left|\frac{bk}{2\pi}\right|^2 d\Omega = \frac{b^2}{\lambda^2} d\Omega \qquad w = 4\pi\frac{b^2}{\lambda^2}$$

1-st Contradiction

Dimensionless probability,
not a cross section

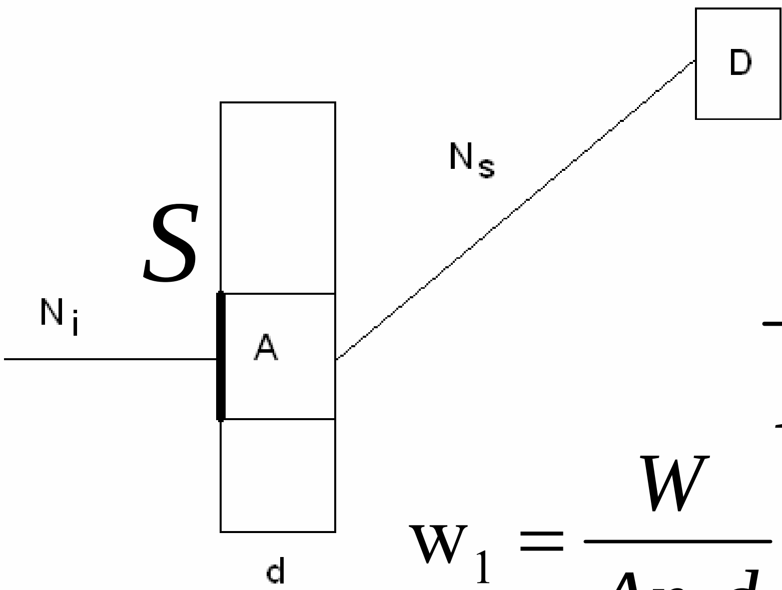
Phenomenological definition of the cross section

$$\sigma_s = \frac{\text{Number of events per unit time}}{\text{Incident flux density}} = \frac{N_s}{J}$$



$$\sigma_{1a} = \sigma_s / N_a = \sigma_s / V n_0$$

How to get cross section



$$W = \frac{N_s}{N_i}$$

$$\frac{W}{N_a} = \mathrm{w}_1 \qquad N_a = An_0 d$$

$$\mathrm{w}_1 = \frac{W}{An_0 d} \qquad A\mathrm{w}_1 = \frac{W}{n_0 d} = \frac{N_s}{N_i n_0 d} = \sigma$$

$$\sigma_{\mathrm{theor}} = A\mathrm{w}_1 = \frac{N_s}{N_i n_0 d} = \sigma_{\mathrm{exp}} \qquad w_1 = |b/\lambda|^2 \, d\Omega$$

V.K.Ignatovich, "Contradictions in scattering theory" Concepts of Physics, V.1 p. 51, 2004.

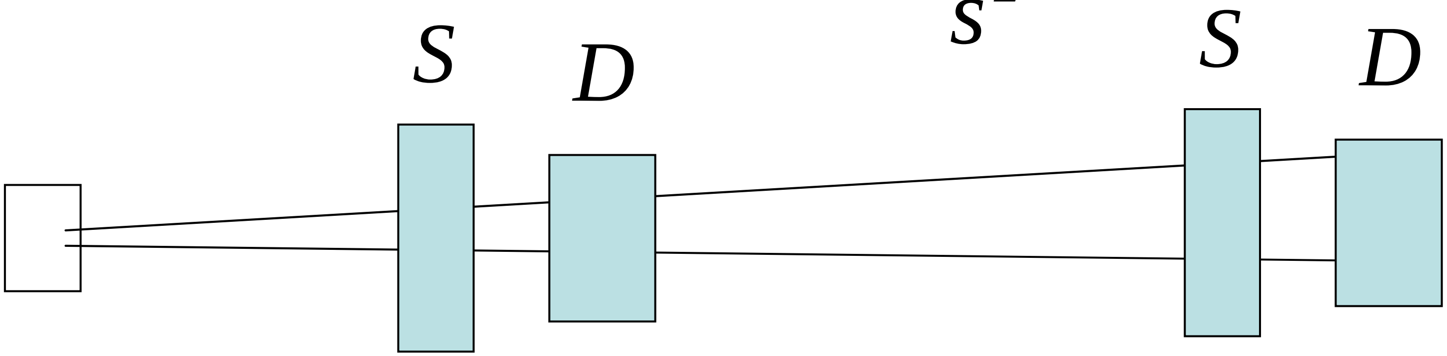
The value of A and its consequence

If neutron is a packet, what this wave packet can be?

$$\varphi(\mathbf{r},\mathbf{k},t,s) = \mathrm{G}(s\,|\,\mathbf{r}-\mathbf{k}t\,|)e^{i\mathbf{kr}-i\omega t} = \int d^3p\, \mathrm{a}(\mathbf{p},\mathbf{k},s)e^{i\mathbf{pr}-i\omega(\mathbf{p},\mathbf{k})t}$$

$$\varphi_G(\mathbf{r},\mathbf{k},t,s) = \left(\frac{s}{\sqrt{\pi}(1+its^2)}\right)^{3/2} e^{i\mathbf{kr}-ik^2t/2}\exp\left(-\frac{s^2(\mathbf{r}-\mathbf{k}t)^2}{2(1+its^2)}\right)$$

$$A = \pi\frac{1+t^2s^4}{s^2}$$



Cross section increases with distance!

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Nonspreading Bessel wavepacket

$$\varphi_B(\mathbf{r},\mathbf{k},t,s) = \exp(ikr - i\omega t)\, j_0(s\,|\,\mathbf{r} - \mathbf{k}t\,|)$$

$$\omega = \frac{k^2}{2} + \frac{s^2}{2}$$

$$A = \frac{\int d^3 r \pi \rho^2 \,|\, j_0(\mathbf{s}\,|\,\mathbf{r}\,|)\,|^2}{\int d^3 r \,|\, j_0(\mathbf{s}\,|\,\mathbf{r}\,|)\,|^2} = \infty$$

The singular de Broglie wave packet

$$\Psi(\mathbf{r},t) = C\,\frac{\exp(-s\,|\,\mathbf{r}-\mathbf{k}t\,|)}{|\,\mathbf{r}-\mathbf{k}t\,|}\,e^{i\mathbf{k}\mathbf{r}-i\omega t}$$

$$A \approx 1/s^2$$

Ultracold neutrons

$$k^2 < u$$

$$u = 4\pi n_o b = k_c^2$$

$$\Psi(\mathbf{r},t) \propto \int d^3p\,\frac{\exp(i\mathbf{p}\mathbf{r}-i\omega(\mathbf{p},\mathbf{k})t)}{(\mathbf{p}-\mathbf{k})^2+s^2}$$

$$\mu \propto \int_{p_\perp > k_c} d^3p\,\frac{C^2}{[(\mathbf{p}-\mathbf{k})^2-s^2]^2} \approx \frac{s}{k_c}$$

Loss coefficient $\mu = 5\cdot 10^{-5}k/k_c \approx s/k_c \Rightarrow s = 5\cdot 10^{-5}k$

$$A \propto 1/k^2 \propto 1/E$$

Does A depend on E or not?

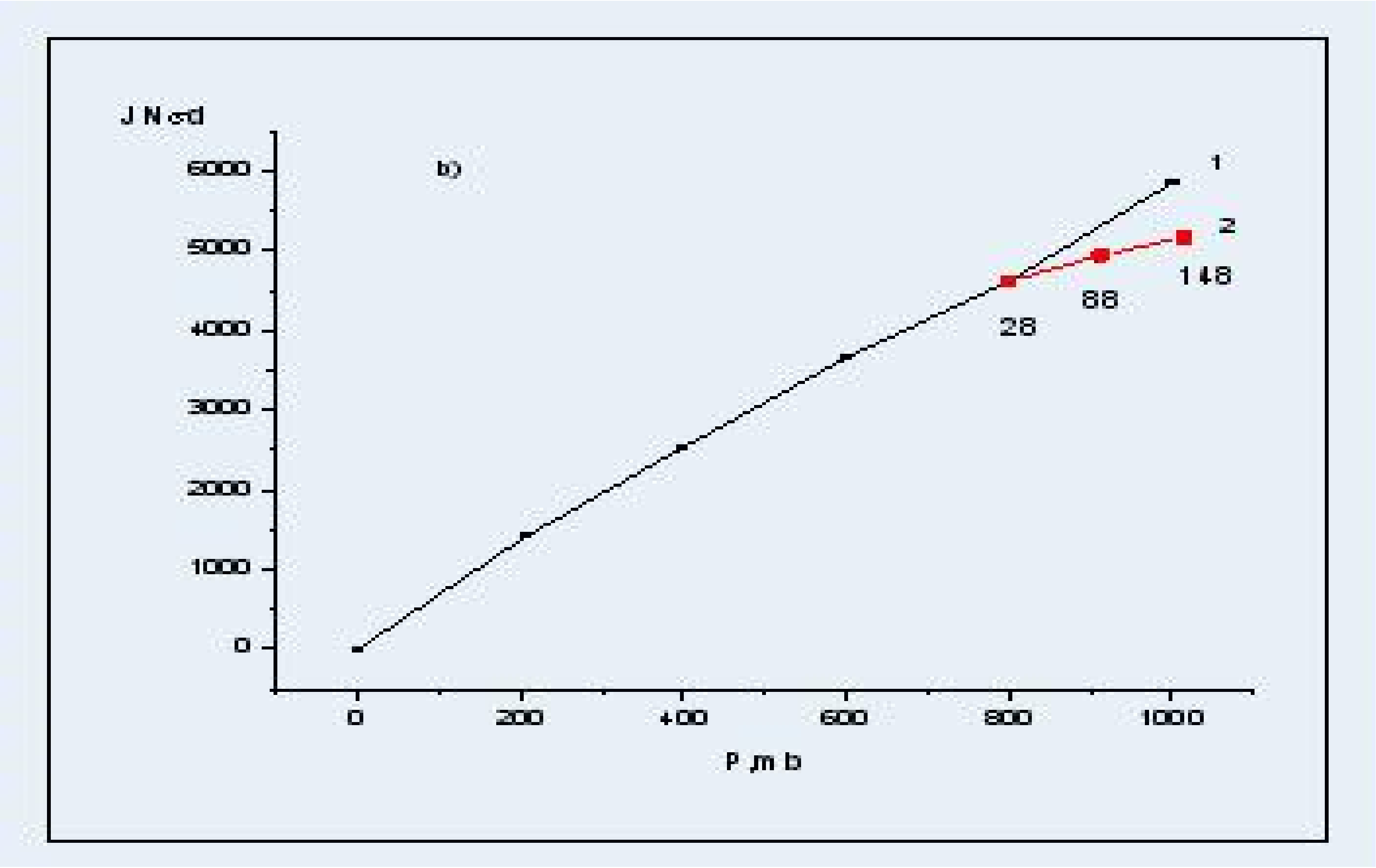
Interaction of neutrons with monatomic gas

$$\sigma = A w_1$$

$$\text{If A} = \text{const, then } \sigma \propto T^{3/2}$$

$$\text{If A} \propto \frac{1}{E}, \text{ then } \sigma \propto T^{1/2}$$

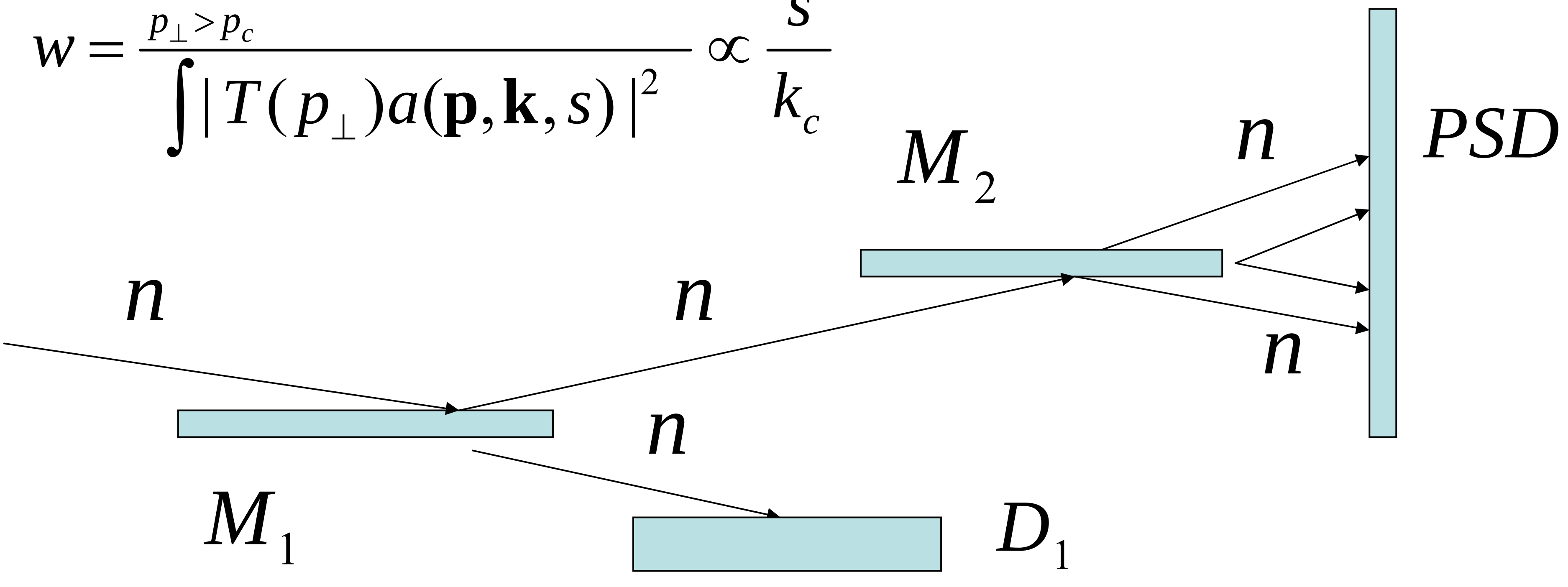
Experimental measurement of σ(**T**) transmission of He in ILL (2003)



Wave packet penetration

$$\hat{T}\varphi(\mathbf{r},\mathbf{k},s,t)=\int T(p_\perp)a(\mathbf{p},\mathbf{k},s)e^{i\mathbf{pr}-i\omega t}$$

$$w=\frac{\int\limits_{p_\perp>p_c}|T(p_\perp)a(\mathbf{p},\mathbf{k},s)|^2}{\int|T(p_\perp)a(\mathbf{p},\mathbf{k},s)|^2}\propto\frac{s}{k_c}$$



The smaller energy the easier is experiment

The first results for neutrons with v=100-200 m/s

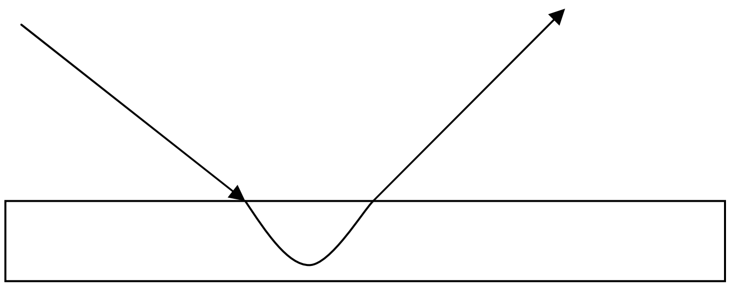
Had shown that A does not depend on E!

Transmission was not found at the level 10^{-4}

M.Utsuro a.o. Proceedings of SPIE v. 3767, pp. 372-379, 1999.

We need to repeat the experiment with remote control of mirrors and absorbers

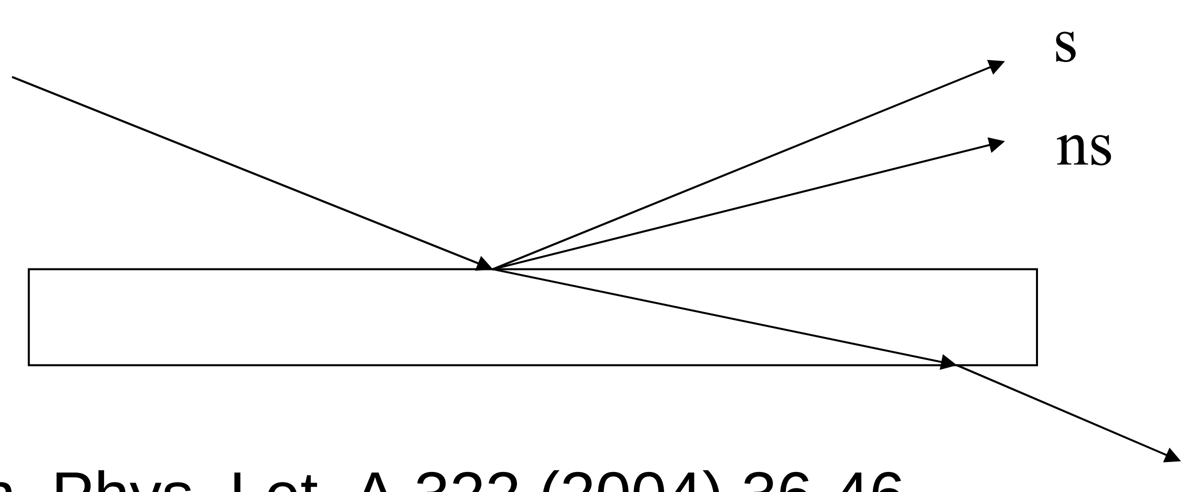
Goos-Hanchen shift



For plane waves there are no space shift. There is only a phase shift. To have the space shift you need wave-packet

For wave packets you have space shift only at total reflection

In the case of nontotal reflection you have nonspecularity



V.K. Ignatovich, Phys. Let. A 322 (2004) 36-46

Nonspecular reflection of a Gaussian w-packet

$$\psi_r(\mathbf{r},t) = \int \exp(i\mathbf{qr}) R(q_\perp) \exp\left(-\frac{(\mathbf{q}-\mathbf{k})^2}{2s^2}\right) d^3q$$

$$R(q_\perp) = \frac{q_\perp - q'_\perp}{q_\perp + q'_\perp} = e^{-2\eta(q_\perp)} \qquad q'_\perp = \sqrt{q_\perp^2 - u}$$

$$\eta(q_\perp) = \mathrm{arcch}\left(\frac{q_\perp}{\sqrt{u}}\right) \approx \eta(k_\perp) + (q_\perp - k_\perp)\frac{\xi}{2}$$

$$\xi = 2\eta'(k_\perp) = 2\frac{d}{dk_\perp}\eta(k_\perp) = \frac{2}{k'_\perp}$$

$$\psi_r(\mathbf{r},t) = R(k_\perp)\int e^{i\mathbf{qr}} \exp\left(-\frac{(\mathbf{q}-\mathbf{k})^2}{2s^2} - (q_\perp - k_\perp)\xi\right) d^3q \Rightarrow$$

$$\Rightarrow R(k_\perp)\exp\left(\frac{s^2\xi^2}{2}\right)\int \exp(iq_\perp z)\exp\left(-\frac{(q_\perp - k_\perp + \xi s^2)^2}{2s^2}\right) dq_\perp =$$

$$= R(k_\perp)\exp\left(\frac{s^2\xi^2}{2}\right)\exp\left(i(k_\perp - \xi s^2)z\right)\exp\left(-\frac{z^2 s^2}{2}\right)$$

$$k_\perp = k\cos\theta \Rightarrow k_\perp - \xi s^2 = k(\cos\theta - \sin\theta\,\delta\theta)$$

$$\delta\theta = \frac{\xi s^2}{k_\parallel} = \frac{2s^2}{k_\parallel k'_\perp} \approx 10^{-8}\frac{k}{k'_\perp}$$

Total reflection of a Gaussian w-packet G-H shift

$$\psi_r(\mathbf{r},t) = \int \exp(i\mathbf{qr}) R(q_\perp) \exp\left(-\frac{(\mathbf{q}-\mathbf{k})^2}{2s^2} \right) d^3q$$

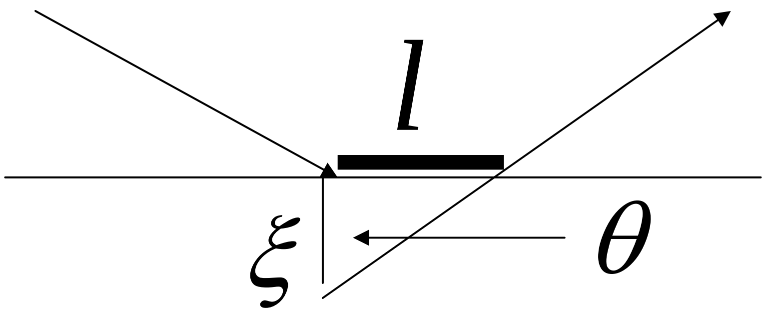
$$R(q_\perp) = \frac{q_\perp - iq''_\perp}{q_\perp + iq''_\perp} = e^{-2i\eta(q_\perp)} \qquad q''_\perp = \sqrt{u - q_\perp^2}$$

$$\eta(q_\perp) = \arccos\left(\frac{q_\perp}{\sqrt{u}} \right) \approx \eta(k_\perp) - (q_\perp - k_\perp)\frac{\xi}{2}$$

$$\xi = -2\eta'(k_\perp) = -2\frac{d}{dk_\perp}\eta(k_\perp) = \frac{2}{k''_\perp}$$

$$\psi_r(\mathbf{r},t) = R(k_\perp)\int e^{i\mathbf{qr}} \exp\left(-\frac{(\mathbf{q}-\mathbf{k})^2}{2s^2} + i(q_\perp - k_\perp)\xi\right) d^3q \Rightarrow$$

$$= R(k_\perp)\exp\left(ik_\perp z\right)\exp\left(-\frac{(z+\xi)^2 s^2}{2}\right)$$



$$l = \xi \,\mathrm{tg}\,\theta \approx \frac{2k}{k_\perp k''_\perp}$$

$$ls \approx 10^{-4}\frac{k^2}{k_\perp k''_\perp}$$

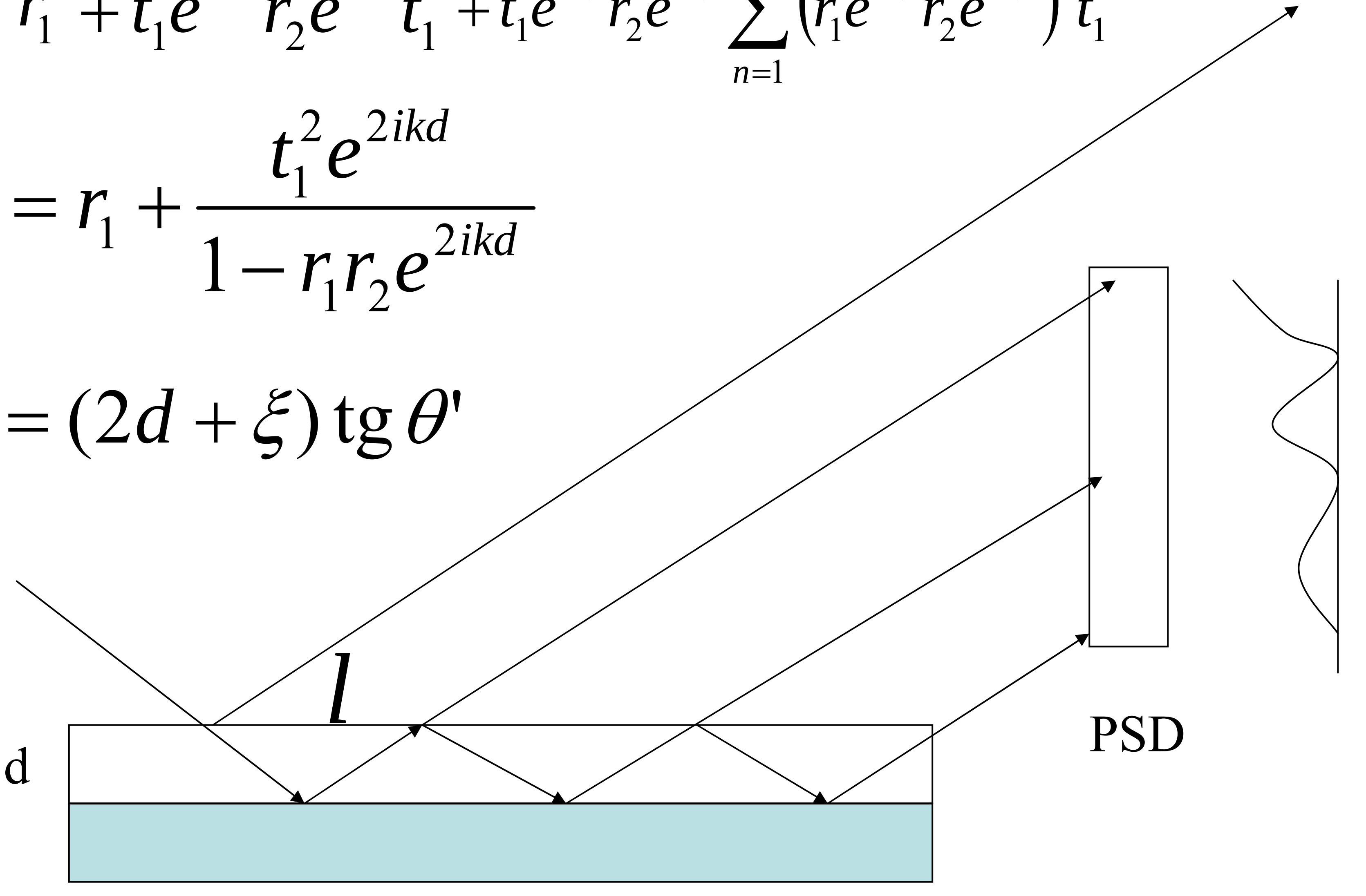
For $\lambda = 6\,\mathring{\mathrm{A}}$ we have $ls \approx 1$

Goos-Hanchen shift analogy

$$R = r_1 + t_1 e^{ikd} r_2 e^{ikd} t_1 + t_1 e^{ikd} r_2 e^{ikd} \sum_{n=1}^{\infty} \left(r_1 e^{ikd} r_2 e^{ikd} \right)^n t_1$$

$$R = r_1 + \frac{t_1^2 e^{2ikd}}{1 - r_1 r_2 e^{2ikd}}$$

$$l = (2d + \xi)\,\mathrm{tg}\,\theta'$$



Conclusion

You don’t believe, but packets do exist!